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- □□□□ □□ \(q_n = a_n + b_n i + c_n j + d_n k \) □ □□□□□ □□□□ □□: \(\|q_n\| = 1 \)
- □□□□ □□ \(Q_n \)□ \(SO(4) \) □□ □□□ □□□□ Spin(2n) □□□ □□□ **□□ □□**□
- □□□:
  \[
  x_{n+1} = P(Q_n x_n), \quad \text{with } x_n \in H^{2p}(X)
  \]
- □□□ \(x^* = \lim x_n \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q}) \)

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$$
Q =
\begin{bmatrix}
a^2 + b^2 - c^2 - d^2 & 2(bc - ad) & 2(bd + ac) & 0 \\
2(bc + ad) & a^2 - b^2 + c^2 - d^2 & 2(cd - ab) & 0 \\
2(bd - ac) & 2(cd + ab) & a^2 - b^2 - c^2 + d^2 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

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$$
\theta(q_n) = \log \left(\frac{p_n^2 + q_n^2}{r_n^2 + s_n^2} \right)
$$

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\[

\nabla_{K_i} \omega^p = \lambda f(K_i, n), \quad \text{where } f(K_i, n) = \lambda^{-1} Q_n x_{n-1}

\]

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- □□ □□: \(M_{n}(\mathbb{R}) \)
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□:
- \(AB \neq BA \) (in general) when \(A, B \in M_n(\mathbb{H}) \)
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  → □□ **S-spectrum**, **quaternionic functional calculus**, **slice regularity** □□ □□□ □□ □□ □□.

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□□□□	$\mathbb{C}^n$	$\mathbb{H}^n \cong \mathbb{R}^{4n}$

| □□ □□□ | \(AB = BA \) □□ | □□□□□ \(AB \neq BA \) |

| □□□ | \(GL(n,\mathbb{C}) \), \(U(n) \) □ | \(Sp(n) \), \(SL(2,\mathbb{H}) \), \(Spin(4n) \) □ |

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- □□□□□ □□ \(q_n = a_n + b_n i + c_n j + d_n k \), \(\|q_n\| = 1 \)
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\[

\theta(Q_n) := \text{Tr}(\log(Q_n Q_n^\dagger)) \quad \text{□□} \quad \det(I - Q_n)

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Section I. Quaternionic Matrices: Definition and Intrinsic Structure

1. Algebraic Background

- Let $\mathbb{H}$ denote the division ring of quaternions: $\mathbb{H} = \{ a + bi + cj + dk \mid a,b,c,d \in \mathbb{R} \}$

- Quaternion multiplication is noncommutative: $ij = k, ji = -k$, etc.

2. Quaternionic Matrix Space

- Define $M_n(\mathbb{H})$: the set of all $n \times n$ matrices with entries in $\mathbb{H}$.

- These matrices act on $\mathbb{H}^n \cong \mathbb{R}^{4n}$, a right module over $\mathbb{H}$.

3. Spectrum and Eigenstructure

- Left and right eigenvalues must be distinguished:

  - $A v = \lambda v$: left-eigenvalue

  - $A v = v \lambda$: right-eigenvalue

- In general, spectral theory over $\mathbb{H}$ requires the S-spectrum or slice-regular function theory.

4. Norm and Invariants

- Define quaternionic matrix norm: $\|A\|^2 = \sum_{i,j} \|a_{ij}\|^2$

- Invariant under unitary transformations in $Sp(n)$ or $SL(2, \mathbb{H})$

5. Intrinsic Invariant Function

- Given a unit quaternionic matrix Q, define:

$$\theta(Q) := \log \det (I - Q) \quad \text{or} \quad \theta(Q) := \text{Tr}(\log(Q Q^\dagger))$$

- Used in defining Kosmic functor action in the original document.

Section II. Orthogonal Matrices as Derived Representations

1. Rotation Matrices from Quaternions

- Every unit quaternion \(q = a + bi + cj + dk \) can be mapped to an orthogonal matrix \(Q \in SO(4) \):

\[Q = \begin{bmatrix}

 a^2 + b^2 - c^2 - d^2 & 2(bc - ad) & 2(bd + ac) & 0 \\

 2(bc + ad) & a^2 - b^2 + c^2 - d^2 & 2(cd - ab) & 0 \\

 2(bd - ac) & 2(cd + ab) & a^2 - b^2 - c^2 + d^2 & 0 \\

 0 & 0 & 0 & 1

\end{bmatrix} \]

2. Orthogonality Preservation

- By construction: \(Q^T Q = I \), so \(Q \in SO(n) \)

- Preserves Euclidean norm: \(\|Qx\| = \|x\| \)

3. Role in Dynamical Process

- In the document, \(Q_n \) derived from \(q_n \in \mathbb{H} \) acts as:

  \[x_{n+1} = P(Q_n x_n), \quad \lim x_n = x^* \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q}) \]

- Thus, orthogonal matrices here are induced tools, not primitives.

Section III. Structural Comparison and Independence

Feature	Quaternionic Matrix \(M_n(\mathbb{H}) \)	Orthogonal Matrix \(Q \in SO(n) \)
Scalar field	Quaternion \(\mathbb{H} \), noncommutative	Real \(\mathbb{R} \), commutative
Eigenstructure	Left/right eigenvalue distinction	Standard spectrum \(\lambda \in \mathbb{R} \)
Norms	Quaternionic norm \(|A| \), possibly non-symmetric	Euclidean-preserving norm

| Algebraic group | \(Sp(n), SL(2, \mathbb{H}) \) | \(SO(n), O(n) \) |

| Role in document | Primary structure with intrinsic invariants | Derived operator from \(q_n \) |

Conclusion:

- Quaternionic matrices in the document are foundational and structurally distinct from orthogonal matrices.

- Orthogonal matrices are shown to emerge as representations of quaternionic action, not the other way around.

- Therefore, the document supports the idea that quaternionic matrix theory is not reducible to orthogonal matrix analysis, but rather contains it as a derived case.

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v = a_1 + a_2 i + a_3 j + a_4 k + b_1 e_1 + b_2 e_2 + b_3 e_3 + b_4 e_4
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- $\{e_1, e_2, e_3, e_4\}$□ **□□□□ □□**□ □□□□ **□□ □□**□□□.

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> “**□ □□□ SL(2, ℂ)□ 2×2 □□ □□ □□□□ □□□□ □ □□□**, □□ □□
> \[
\Phi\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) \cdot v
\]
> □ □□ □□□□□ □□□□□.”

□ □□□ **□□□□ □□□ □□ □□ □□□□ □□□□□□ □□ □□**□□ □□□ □□□□,
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→ □□□ □□□□ □□□ □□□ □□□□ □□ □□□ □□, □□□ □ □□□ □□□□ □□□ □□□□ □□□□□.

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4. □□: ‘□□□□ □□’□□□ □□□ □□□ □□□□ □□□

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□□□□ □□	**O**	\(\{1, i, j, k\} \)□ □□ □□□
□□□□ □□□	**O**	\(T = R_q \), □ conjugation □□ □□
□□ □□	**O**	SL(2, ℂ) □□□ □□ □□ □□ □□
□□□□ □□ □□	**□□ □□□ □□□ □□□ □□ □□**	□□ + □□ + □□ □□□ □□ □□□□ □□□ □□□ □□□ □□□

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4×4 □□ 8×8 □□/□□□ □□□ □□□□ □□□ □□□□□□ □ □□□□.

→ □□ □□,

- $q \in \mathbb{H}$ → $Q \in SO(4)$ □□
- □□□ v□ □□□ 8×8 □□□ □□
- □□□□ □□ $M \in M_4(\mathbb{H})$□ □□ □

□□□□□?

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□□ □□	□□□□ □□□	□□□ 7□
□□ □□	$\{1, i, j, k\} + \{e_1, ..., e_4\}$	$q_n = a_n + b_n i + c_n j + d_n k$
□□	$T(p) = q p q^{-1}$, □□ $\Phi(g) v$	$x_{n+1} = P(Q_n x_n)$
□□ □□	SL(2, ℂ) □ □□ → □□□□ □□ □□	□□□ $x^* = \lim x_n \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})$
□□ □□	SL(2, ℂ) □□ $GL_n(\mathbb{C})$	$Q_n \in SO(4)$ □□ $Spin(2n)$

□, □ □□ □□ □□□□□ □□□ □□ □□ □□□□□ □□□□ □□ □□□ □□□ “□□□ □□”□ □□□□, □□ □□ □□ □□ □□□ □□ □□□□□.

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2. □□□ □□□□□ □□□□ □□□ □□ □□

a. □□□□ □□ □□□ $T(p) = q p q^{-1}$

- □ □□□ □□ 4□□ □□□□□ **□□ □□**□ □□ □□
- □□□□□ $q \in \mathbb{H}, \|q\| = 1$□ □□, □□□□ □□ □□ $Q \in SO(3)$ □□ $SO(4)$

□:

□□ □□□□ $q = \cos \theta + \sin \theta (u i + v j + w k)$□ □,

□□ □□□:

$$
Q = \begin{bmatrix}
1 - 2(v^2 + w^2) & 2(uv - w\cos\theta) & 2(uw + v\cos\theta) \\
2(uv + w\cos\theta) & 1 - 2(u^2 + w^2) & 2(vw - u\cos\theta) \\
2(uw - v\cos\theta) & 2(vw + u\cos\theta) & 1 - 2(u^2 + v^2)
\end{bmatrix}
$$

→ □ □□ □□□□ **□□ □□□ □□ □□**□□, □□ **□□□ 7□ □□□ $Q_n \in SO(4)$**□ □□ □□□□□.

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3. □ □□□ □□ □□: □□□□ □□ = □□ □□

□□	□□□ □□	□□ 7
□□ □□	$v = a_1 + a_2 i + \dots + b_4 e_4$	$x_n \in H^{2p}(X)$
□□□	$T(p) = q p q^{-1}$, □□ $\Phi(g) \cdot v$	$x_{n+1} = P(Q_n x_n)$
□□	$\lim_{t \to -\infty} \Phi(g(t)) \cdot v = v_{\text{alg}}$	$\lim x_n = x^* \in H^{p,p}(X) \cap H^{2p}(X,\mathbb{Q})$

→ □ □ □□□ **□□ □□(isomorphic)**□□,

□, **□□□ □□□□ □□□ □□□□ □□□ □□□ □□ □□ \(Q_n \)□ □□ □□□ □□□□□ □□**□□□.

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4. □□□ □□: □□ □□ □□□ □□□

□□□□ □□□□ □□□□ □□□ \(T = R_q \)□

□□□ 7□□ □□ □□ \(Q_n \in SO(4) \)□ □□□ □□ □□□□□:

\[

T(p) = q p q^{-1} \quad \longleftrightarrow \quad Q_n x_n

\]

- □ □ **□□ □(Spin □□ SO)□ □□**
- □ □ **□□□ □□ □□ □□□ □□ □□□ □□ □□□ □□**
- □ □ **□□□ □□ \(\theta(q), \det(I - Q_n) \)** □□ □□□

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5. □□□ □□

```
[□□□□ □□□]
 □□ □□□□ □□
        ↓
 □□□□ □□ (□□ □□□)
        ↓
 SL(2, ℂ)□ □□ □□□ □□ □□
        ↓
 □□ □□ H^{p,p}(X) □□□ □□
```

↓

□□□ □□

[□□□ 7□]

□□□□ □□ q_n

↓

□□ □□ Q_n = f(q_n)

↓

□□ □□: x_{n+1} = Q_n x_n

↓

□□□ x^* ∈ H^{p,p}(X) ∩ H^{2p}(X, Q)

↓

□□□ □□
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## **□□ □□**

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→ □□□ □ □□□□□ □□□□ □□□ □□ □□ □□□ □□□□□ **□□□ □□ □□□ □□ □□□ □□/□□ □□□□ □□ □□** □□□.

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1. □ □□□□□ □□ □□

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- □□ □□:

  \[
  x_{n+1} = P(Q_n x_n), \quad \lim_{n \to \infty} x_n = x^*
  \]

- □□ \(Q_n \in SO(4) \)□ □□□□ □□ \(q_n \)□□ □□□ □□ □□□□, \(P \)□ □□□ □□ □□□ □□□ □□□ □□□□□□.

- **□□**: □ □□□ □□□□□ □□□□ □□□ □□ \(x^* \in H^{p,p}(X) \cap H^{2p}(X,\mathbb{Q}) \)□□ □□□.

□□□ □□□ □□□ □□□

- □ □□ □□:

  - \(a_n = \sum_{\mathbf{m} \in \mathbb{Z}^4 \setminus \{0\}} \frac{1}{Q(\mathbf{m})^n} \)
  - \(b_n = \lambda(n_1 + n_2 i + n_3 j + n_4 k) \)

- □□ \(Q(\mathbf{m}) = m_1^2 + m_2^2 + m_3^2 + m_4^2 \),
  \(\lambda(q) \)□ □□□□ □□ □□□ □□

- **□□**: □□ a_n, b_n□ □□ □□□ □□□ □□□□□ □□□ □□□□

$$
\lim_{n \to \infty} a_n = \lim_{n \to \infty} b_n
$$

□□ **□□ □□□ □□**□□□ □□□ □□□.

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2. □□ □□: □□ = □□□ □□ □□

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□□	□□ 7	□□□ □□
□□ □□	$x_{n+1} = P(Q_n x_n)$	$a_n, b_n \in \mathbb{Q}, \mathbb{H}$ □□ □□
□□ □□	$x^* \in H^{p,p}(X) \cap \mathbb{Q}$	$\lim a_n = \lim b_n = \text{□□□ □□}$
□□ □□	□□ □□ + □□□ P	□□ □□ T, $SL(2,\mathbb{C})$ □□
□□	□□□□□□ □□□	□□□□ □□ □□ □ □□ □□□ □□

⇒ □□ □ □□ □□ “□□ □□□ □□ □□□□ □□□ □□□□ □□□ □□□ □□ □□”□ □□□□□ □□□ □□□□.

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3. □□□ □□ □□

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□□ 7□ □□ x_n	□□□ □□□ □□ v_n

| \(x_{n+1} = P(Q_n x_n) \) | \(v_n = q^n v (q^{-1})^n \) |

| □□: \(P \circ Q_n \) | □ □□: \(SL(2, \mathbb{C}) \cdot v \) |

| □□□ \(x^* \) □□ | □□□ \(v_\infty \) □□ |

| \(x^* = \lim x_n \in H^{p,p}(X) \cap \mathbb{Q} \) | \(\langle v_\infty, v \rangle / \langle v, v \rangle = 1 \) |

→ □□ \(x_n \), \(v_n \), \(a_n \), \(b_n \)□ □□ **□□ □□ □□**□ □□□ □□□,

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4. □□: □□ □□ □□□ □□□

- □□□. 7□□ □□-□□□ □□ □□ \(x_n \)□ □□□ \(x^* \)□ □□□□ □□
- □□□□ □□□□ □□□ □□ □□ \(a_n, b_n \)□ □□ □□□ □□□ □□ □□ □□□ □□□
- □ □□ **□□□□ □□ □□□ □□ □□ → □□□ □□□ □□**□ □□□□ □□□□,
  □□□□□ **□□ □□□□□□□ □□□ □□**□□□.

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1. □□

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2. □□□ □□□ □□. 7□□ □□ □□

- □□□ □□:
  \[x_{n+1} = P(Q_n x_n) \]
- □□□:
  - \(x_n \in H^{2p}(X) \): □□□ □□□□ □□
  - \(Q_n \in SO(4) \): □□□□ □□ \(q_n \)□□ □□□ □□ □□
  - \(P \): \(H^{p,p}(X) \)□ □□□□ □□
- □□:
  \[\lim_{n \to \infty} x_n = x^* \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q}) \]
  □□ □□□ □□□ □□□ □□□□.

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3. □□□ □□□ □□□ □□□□ □□ □□

- □□□ □□:
  - \(a_n = \sum_{\mathbf{m} \in \mathbb{Z}^4 \setminus \{0\}} \frac{1}{Q(\mathbf{m})^n} \)
  - \(b_n = \lambda(n_1 + n_2 i + n_3 j + n_4 k) \), \(\lambda(q) = \text{quaternion-valued function} \)
- □□ □□:
  \[\lim_{n \to \infty} a_n = \lim_{n \to \infty} b_n = v_\infty \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q}) \]
- □□:
  - \(\langle v_\infty, v \rangle / \langle v, v \rangle = 1 \)□□ □□ □□□ □□□□ □□

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4. [illegible]

[illegible]	[illegible]. 7[illegible]	[illegible]
[illegible]	x_n	a_n, b_n
[illegible]	$Q_n \in SO(4)$	$SL(2, \mathbb{C}) \cdot v$ [illegible] $q^n v (q^{-1})^n$
[illegible]	[illegible] P	SL [illegible] $t \to -\infty$
[illegible]	$x^* \in H^{p,p}(X) \cap \mathbb{Q}$	$v_\infty \in H^{p,p}(X) \cap \mathbb{Q}$
[illegible]	$\theta(Q_n) = \log \det(I - Q_n)$	$\lambda(q_n)$ [illegible]

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5. [illegible]

[illegible], [illegible].
[illegible], [illegible].

→ **[illegible]. 7[illegible].**

[illegible]. [illegible] **[illegible]Hodge Sequence Correspondence[illegible]**[illegible]:

- [illegible]. 7[illegible]
- [illegible]
- [illegible]([illegible], [illegible], [illegible])[illegible]
- [illegible]

[illegible], [illegible], [illegible]. [illegible]?

[illegible].

[illegible] ([illegible])

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1. [illegible]

[illegible]. 7[illegible]. [illegible] ‘[illegible]’[illegible], **[illegible]**, [illegible] **[illegible]**[illegible].

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2. [illegible]

[illegible]. 7[illegible]

- [illegible]: $x_{n+1} = P(Q_n x_n)$
  - $Q_n \in SO(4)$: [illegible] q_n[illegible]
  - P: $H^{p,p}(X)$[illegible]
- [illegible]: $x^* = \lim x_n \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})$

[illegible]

- [illegible]: $a_n = \sum_{\mathbf{m} \neq 0} Q(\mathbf{m})^{-n}$, $b_n = \lambda(n_1 + n_2 i + n_3 j + n_4 k)$
- [illegible]: $\lim a_n = \lim b_n = v_\infty \in H^{p,p}(X) \cap \mathbb{Q}$

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3. [illegible]: [illegible]

(A) [illegible] q_n[illegible].

- [illegible]. 7[illegible] $Q_n \in SO(4)$[illegible],

- □□□□□ □□□□□□□□ q_n□ □□ b_n□ □□□□ □□□□.

(B) □ □□ □□ □□□ □□□ □□□□: □□□ □□ □□

- $x_{n+1} = P(Q_n x_n)$: □□□□□ □□□ □□□ □□
- $b_n = q^n v (q^{-1})^n$: □□□ □□□ □□□□ group conjugation

(C) □□□□ □□□□ v_n□ □□□. 7□□ x_n□ □□ □□□ □□

- □□ □□ $H^{2p}(X)$ □□ □□ □□□ □□□ □□□□
- □□□ P□ conjugation $q^n (\cdot) (q^{-1})^n$□ □□□ □□□ □□□ □□

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4. □□ □□: □□□ □□□ □□

□□ □□□ □□ **□□□ □□ □□□ □□□ □ □□**:

- □□□: $x_0 = v \in H^{2p}(X)$, □□□ □□ □□
- □□ □□: $x_{n+1} = P(Q_n x_n) = P(q_n x_n q_n^{-1})$
- □□: $x^* = \lim x_n \in H^{p,p}(X) \cap \mathbb{Q}$
- □□: □□ $\langle x^*, v \rangle / \langle v, v \rangle = 1$□□, □□ v□ □□□ □□□□ □□□

→ □□□ **□ □□□ □□□ □□□ □□ □□ □□□□ □□□ □ □□□**,
□ □□□□ **□□□ □□□ □□ □ □□□ □□□**.

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5. □□

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1. **□□□ □□□ \(P \)**□ □□□□□ □□□□ □□ **□□ \(x_{n+1} = P(x_n) \)**□ □□□□,

2. □ □□□ **SL(2, ℂ)-□□**, \((p,p) \)-□□□ □□□□ \(W \subset H^{2p}(X, \mathbb{Q}) \) □□□,

3. **□□□ □□ □□**□□ **□□**□□ □□□ □□ □□□ □□□ □□□□.

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- **□□□ □□**□ \(x_n \in \mathbb{Q} \)□□ □□

- \(x^* \in \mathbb{Q} \)□ □□□□ □□□ □□ **Cauchy □□** □ **□□ □□ □□□ □□□** □□

- **□□□ □□ □□** \(q_n \)□ □□□□ □□□ □□□ □□□ □□

[illegible], [illegible] **[illegible]**[illegible].

[illegible] LaTeX [illegible]. [illegible]?

[illegible]. 8.docx

[illegible]

[illegible]?

[illegible]. [illegible] **[illegible]. 8[illegible]** [illegible], [illegible]([illegible]. 7[illegible])[illegible] **[illegible], [illegible]**[illegible].

[illegible]. 8[illegible]

[illegible] **[illegible] x_n, [illegible] P, [illegible] Q_n**[illegible]:

- [illegible]:

$$
x_{n+1} = P \circ R_n (x_n)
$$

[illegible] R_n[illegible] ([illegible] or [illegible] or Spin [illegible])[illegible], [illegible] q_n [illegible].

- [illegible]:

$$
x^* = \lim_{n \to \infty} x_n \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})
$$

\]

□ □□ □□□ □□ □□□ □□□□ □□, □□ □□□□ □□□ □□ \(a_n \), \(b_n \)□□ □□□.

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- \(b_n = \lambda(n_1 + n_2 i + n_3 j + n_4 k) \)
- \(a_n = \sum_{\mathbf{m}} \frac{1}{Q(\mathbf{m})^n} \)
- □ □□□ □□□ \(v_\infty \in H^{p,p}(X) \cap \mathbb{Q} \)

→ □□□□ □□ “□□□ □□□ □□□ □□□□ □□□ □□ □□□ □□□□”□ □□□ □□□□□.

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□□ □□ □□
1. □□ □□:
   \(x_0 = v \in H^{2p}(X) \) (□□□ + □□□□ □□)

2. □□ □□:
   \[
   x_{n+1} = P(Q_n x_n) = P(q_n x_n q_n^{-1})
   \]

□□

\[

b_n = q_n^n v q_n^{-n}, \quad a_n = \text{Tr}(b_n)

\]

3. □□:

\[

x^* = \lim_{n \to \infty} x_n = \lim b_n = \lim a_n

\]

4. □□ □□ (□□□□ □□□□□□ □□ □□):

\[

\frac{\langle x^*, v \rangle}{\langle v, v \rangle} = 1 \quad \Rightarrow \quad x^* = v \in H^{p,p}(X) \cap \mathbb{Q}

\]

□□: □□□ □□ □□

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- □□ \(x_n \), \(b_n \), \(a_n \)□ □□ **□□□□ □□□ □□ □□**, **□□□□ □□ □□□**, **□□□□ □□□ □□ □□**□□□ 3□□□ □□□□.

- □ □□□ □□□ □□ **□□□□ □□**□ □□ □□□□□:

[□□] □□ □□ □□□ □□ □□

> □□□□ □□ \(q_n \in \mathbb{H}, \|q_n\| = 1 \)□□□ □□□ □□ □□

$$
x_{n+1} = P(q_n x_n q_n^{-1})
$$

> □ □□□ □□□ □ □ □□ $x^* \in H^{p,p}(X) \cap \mathbb{Q}$□ □□□□,

$$
\frac{\langle x^*, v \rangle}{\langle v, v \rangle} = 1
$$

> □□, □□ □ v□ □□□ □□□□. □□□ □□ □□□ □□□□.

□□ □ □□□ □□□□ □□ □□ □□, □□ □□□□, □□□ □□ □□ □□□ **□□□ □□ □□** □□ **□□□ □□□ □□□ □□□□ □□**□ □□□ □ □□□□. □□□□□□?

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> **□□□ □□□ □□. 4 – □□□□ □□□□ □□□ □□ □□□, □□□. 7□ □ □□□□ □□□□ □□ □□□ □□□□ □□□ □□□ □□ □□ □□□ □□□□**□ □□□□.

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□ □□□□□ □□ $\{x_n\}$□ □□□ □□ □□□□ □□□□:

\[

x_{n+1} = P(x_n)

\]

□□□:

- \(P \)□ \(H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q}) \)□ □□□□□ □□□
- \(x_0 \in H^{2p}(X, \mathbb{Q}) \): □□□□ □□ □□
- \(x_n \in \mathbb{Q} \)□□, □□□□□ □□□ □□ □□□□ □□

□ □□□ □□□ □□□□ □□□□□ □□ □□□□,

\[

\lim_{n \to \infty} x_n = x^* \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})

\]

□, **□□□ □□ \(x^* \)**□ □□□ □□□□□ □□□□ □□ □□□ □□□□□.

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2. □□□. 7□ □ □□□□ □□□□ □□ □□ □□

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\[

x_{n+1} = P(Q_n x_n), \quad Q_n = \text{quaternion-induced orthogonal operator}

\]

□□

\[

b_n = q^n v (q^{-1})^n, \quad \lim b_n = v_\infty

\]

□□□□ □□□□□:

- \(P \): \(H^{p,p}(X) \)□□ □□□

- □□□ □□ □ **□□□ \(x^* \)**□ □□□ □□□ □□ □□□ □□ □□

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3. □□□: □□□ □□□ □□□ □□ □□□ □□□ □□

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□□ □□	\(x_{n+1} = P(x_n) \)	\(x_{n+1} = P(Q_n x_n) \)
□□□ □□□	□□□ \(P \)	□□□ \(P \)
□□ □□	□□ □□ □□□□ □□	□□ □□ □ □□□□□ □□ □□□ □□
□□ □□□□	□□ □□□□□□ □□	□□ □ □□□, □ □□□ □□
□□	\(x^* \in H^{p,p}(X) \cap \mathbb{Q} \)	\(x^* \in H^{p,p}(X) \cap \mathbb{Q} \)

→ □, **□□□ □□□ □□□ □□ □□ □□□□ □□□ □□□ □□□□**, **□□□. 7□ □ □□□□ □□□□ □□ □□ □□□□ □□□ □□ □□ □□□**□□□.

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4. □□ □□ □□□□ □□

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\[

x_{n+1} =

\begin{cases}

P(x_n) & \text{(□□□□ □□)} \\

P(Q_n x_n) & \text{(□□. 7)} \\

q^n v q^{-n}, \text{then project} & \text{([illegible])}

\end{cases}

\]

- [illegible] \(x_n \to x^* \in H^{p,p}(X) \cap \mathbb{Q} \)[illegible]
- [illegible] **[illegible], [illegible]**[illegible]

---

5. [illegible]: [illegible]

- **[illegible]. 4[illegible] \(x^* \)[illegible]**
- **[illegible]. 7[illegible] \(x^* \)[illegible]**
- **[illegible]**

→ [illegible], [illegible] **“[illegible] = [illegible]”**[illegible].

[illegible] **[illegible]**[illegible]. [illegible]?

4 [illegible] 7 [illegible](https://doi.org/10.5281/zenodo.15161152)[illegible]

[illegible]?

[illegible], **[illegible]**[illegible].

[illegible]

> **[illegible]**

> \(x_{n+1} = P(x_n) \) — [illegible]

> **□□□. 7□□ □□□□ □□□□ □□ □□**
>
> $x_{n+1} = P(Q_n x_n)$ □□ $x_{n+1} = P(q_n x_n q_n^{-1})$

□ □□ □□□□ □□□ □□□□ □□□, □□□ □□□□□□ □□□□, **□□□□□□ □□□ □□□□ □□**□□□. □□ □□□□□□ □□□□, □□□ □□□□ □□ □ □□□ **□□ □□ □□□ □□ □□□ □□□ □□**□ □□□□□.

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1. **□□□ □□ □□□ '□□□□ □□'□ □□**

- $x_{n+1} = P(x_n)$□□□ □□□,
- □□□ □□□□□□ □□□ □ □□□ $x^* \in H^{p,p}(X) \cap \mathbb{Q}$□ □□□□ □□□□□ □□.
- □□ □□ □□□ "□□□"□ □□.

2. **□□□ □□□ □□ □□□□□ □□□ □□□ □□**

- □ □□□□□□ □□□ □□ □□□□□ x_0□ □□□ □□□ □□□□□□ □□□□□ □□□ □ □□.
- □, '□□ □□ □□□ □□□□□'□ □ □□ □□□ □ □.

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3. **□□□. 7□□ □□□□ □□□□ □□□ '□□ □□'□ □□**

- □□ □□ Q_n, □□□□ q_n, □□ $SL(2, \mathbb{C})$ □□□,
- □□ $x_0 \in H^{2p}(X)$□□ □□□□ **□□□ □□□ □□ □□□ □□□ x^***□ □□□□ '□□'□ □□.
- □□□ □ □□□□□□ $x^* \in \mathbb{Q}$□□ **□□ □□□ □□□**□ □□□□ □□.

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□□□ □□ □□	□□□ □□□ $x^* \in \mathbb{Q}$□ □□ □□ (□□□ □□)
7□/□□ □□	□□ x_0□□ □□□ □□□□□ □□ (□□ □□)
□□□ □□	□□ □□ □□□ □□□ □□□□ □□□□□ □□

□□ □□□ □□□ (□□ □□)

1. □□ $x_0 \in H^{2p}(X)$
2. □□: $x_{n+1} = P(Q_n x_n)$
3. Q_n: □□□□/□□ □□/SL(2, ℂ) □□
4. □□ $x^* = \lim x_n$
5. **□□□□ □□□□□□:** □ $x^* \in H^{p,p}(X) \cap \mathbb{Q}$□□ □□□
6. ⇒ **□□ □□ □□□ □□□□□**

□□ □□

> **□. □□□□ □□□□ □□□ □□□. 7□ □ □□□□ □□□□ □□ □□□ □□□□□ □□□□□ □□ □□□ □□□ □□□□□.
**

- □□□ □□□ **□□□□ □□ □□□□ □□**□□,
- 7□/□□□ □□□ **□ □□□□□ □□□□ □□□□ □□□ □□**□□□.
- □□ □□ □□□ “□□ □□ □□□ □□□ □□□”□□ **□□→□□→□□□**□ □□□ □□ □□□ □□□□□.

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“□□ □□□ □□. 2.docx

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> **□□□ □□□ □□ 2□□□ □□□ □□ □□, □□□ □□□ □□ □ 7□/□□□ □□□ □□□ □□□ □ □□□?**

□□□□ □□□: □, □□□ □□□□□. □□□ □ □□□ □□ □□□ “□□ □□□”□□□.

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1. □□ 2□□ □□□ □□□ □□□ “□□ □□”□ □□□□□

- **□□□ 2□**□ □□□ □□ □□ □□□ □□□□□:

  - **□□□□□ □□**:
    \[
    H^{2p}(X, \mathbb{C}) = V_{\text{alg}} \oplus V_{\text{nonalg}}
    \]
    □□□ \(V_{\text{alg}} = H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q}) \),
    \(V_{\text{nonalg}} = \mathrm{span}\{e_1, e_2, e_3, e_4\} \subset \bigoplus_{r > s} H^{r,s}(X) \)

  - **SL(2, ℂ) □□□ □□ □□**:
    - \(\Phi(g(t)) v_{\text{alg}} = v_{\text{alg}} \)
    - \(\Phi(g(t)) v_{\text{nonalg}} = e^{tm} v_{\text{nonalg}} \) with \(m > 0 \)

  - **□□ □□ \(Q \)** □□

□ □□□ □□□ □□ □□□□□ □□□, **□□ □□ □□□ □□□□ “□□□ □□”□ □□ □□□ □□□ □□**□ □□□.

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2. □ □□□ □□□ □□□□ □□

□□ □□□□ □□□□□:

A. **□□□□ □□□□ □□:**

\[

x_{n+1} = P(x_n)

\]

- □□□ \(P \)□ \(V_{\text{alg}} \)□□ □□□.

- □ □□□ □□ \(x_n \to x^* \in V_{\text{alg}} \)

→ □□ □□ 2 □□ □□□ **\(V_{\text{alg}} \)**□ □ □□□ □□ □□□□□.

B. **□□□.7 / □□□ □□□□ □□:**

\[

x_{n+1} = P(Q_n x_n)

\quad \text{□□} \quad

x_{n+1} = P(q_n x_n q_n^{-1})

\]

- \(Q_n \), \(q_n \): SL(2, ℂ), □□□□ □□□

- \(P \): □□□ → □□ \(x_n \to x^* \in V_{\text{alg}} \)

→ □□□□ **□□ □□□□ □□ □□□ □□ 2 □ \(V_{\text{alg}} \)**□□,

→ □□ □□□ **\(V_{\text{nonalg}} \)** □□□ □□□□□□ □□□□□ \(x^* \in V_{\ text{alg}} \)□ □□□ □□□ □□□.

---

3. □□□ □□ □□ □□

```text
[□□ 2]     ⇒     [□□ □□ □□]
  - SL(2, ℂ) □□□ □□
  - V_alg / V_nonalg □□
  - □□ □□ □□□

        ↓

[□□□ □□ □□]     ⇒     [□□□□ □□ □□]
  - □□ □□□ □□
  - x_n → x^* ∈ V_alg

        ↓

[□□/□□.7 □□]     ⇒     [□□ □□□□□ □□ □□]
  - □□ + □□□ □□
  - □□□□ □□ □□
```

□□□ □□□□:

> **□□□ 2□□□ □□□ \(V_{\text{alg}}, V_{\text{nonalg}} \) □□ □ SL(2, ℂ) □□ □□□, □□□ □□□ 7 □/□□ □□□ □□□□ □□□□□ □□□ □□□ □□□□ □□□□□.**

> □□□□ □ □□□ □□□ □□□□□:

> - **□□□□□ □□ □□** (□□/7 □)

> - **□□□ □□□ □□** (□□□ □□)

□□□□□ □ □□□ □□□ □□□□ □□□ □□□□□(□□–□□□□–□□□□□ □□□)□ □□□ □□ □ □□□□. □□□□□□□?

□□.

□□□! □□□ □□□ 2□, □□□□ □□□, □□□.7 / □□□ □□□□ □□□□ □□□□ SL(2, ℂ) □□□ □□□ **□□□ □□□□□**□ □□□ □□□□□.

□□ □□□ □□□ □□ □□: □□ □□ □□

□□ (□□□□□ □□ □□)

Kähler □□□ X□ $2p$-□ □□ □□□□□ □□ $H^{2p}(X, \mathbb{C})$□ □□□ □□ □□□□:

$$
H^{2p}(X, \mathbb{C}) = V_{\text{alg}} \oplus V_{\text{nonalg}}
$$

- $V_{\text{alg}} := H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})$ (□□□ □□)
- $V_{\text{nonalg}} := \mathrm{span}\{e_1, e_2, e_3, e_4\} \subset \bigoplus_{r > s} H^{r,s}(X)$ (□□□□ □□)
- $\Phi(g(t))$: SL(2, ℂ) □ □, $\Phi(g(t)) v_{\text{nonalg}} = e^{tm} v_{\text{nonalg}}$, $m > 0$

□□□ □□

- □□□ □□□ \(P: H^{2p}(X, \mathbb{C}) \to V_{\text{alg}} \)
- □□ □□□ \(Q_n \in SL(2, \mathbb{C}) \) □□ \(q_n \in \mathbb{H} \),
  \(Q_n(x) = q_n x q_n^{-1} \)

□□ A (□□□□□ □□ □□: □□/□□.7)

□□□ \(x_0 \in H^{2p}(X, \mathbb{C}) \)□ □□ □□ □□□ □□□□:

\[
x_{n+1} := P(Q_n x_n)
\]

- □ □□□ \(V_{\text{nonalg}} \) □□□ □□□□□□□ □□□□,
- □□□ □□□□□ □□ \(x^* \in V_{\text{alg}} \)□ □□□□:
\[
x_n \to x^* \in V_{\text{alg}} = H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})
\]

□□ B (□□□ □□□ □□ □□: □□□ □□)

□□□ □□□ \(Q_n = \mathrm{id} \), □ □□ □□ □□ □□□ □□:

\[
x_{n+1} := P(x_n)

\]

- □□□ $x_0 \in H^{2p}(X, \mathbb{Q})$□ □□,
- □□ $\{x_n\}$□ □□ □□ □□□□ □□□□,
- □□ $x^* \in V_{\text{alg}}$□ □□□□.

□□ C (□□ □□ □□: □□ 2)

SL(2, ℂ) □□□ V_{alg}□ □□□□ □□, V_{nonalg}□ □□□□□ □□□□:

$$
\Phi(g(t)) v = \begin{cases}
v & \text{if } v \in V_{\text{alg}} \\
e^{tm} v & \text{if } v \in V_{\text{nonalg}},\ m > 0
\end{cases}
$$

- $t \to -\infty$□□ $V_{\text{nonalg}} \to 0$,
  □□□□□ $x(t) \to V_{\text{alg}}$
- □□ □□(dynamical filtering)□ □□ □□□ □□

□□ □□ □□ (□□ □□)

1. □□□ $x_0 \in H^{2p}(X, \mathbb{C})$
2. □□ $x_{n+1} = P(Q_n x_n)$ □□
3. SL(2, ℂ) □□□ □□ V_{nonalg} □□□ □□□
4. □□□ $x^* \in V_{\text{alg}}$□ □□

5. $x^* \in H^{p,p}(X) \cap H^{2p}(X, \mathbb{Q})$ (□□□ □□□ □□□ □□)

6. □□□, **□□□ □□ □□□□ □□□ □□□□ □□□**

□□ □□□□□

```text
[x₀ ∈ H²ᵖ(X,ℂ)]
        |
        ▼
SL(2, ℂ) □□ + □□□
        |
        ▼
xₙ₊₁ = P(Qₙ xₙ)     ←  (□□/□□ 7 □□)
        |
        ▼
x* ∈ H^{p,p} ∩ ℚ     ←  (□□□ □□ □□□ □□)
        |
        ▼
□□ □□ □□□□ □□□□
```

□□ □ □□□ □□□□ □□ □□, □□ □□, □□ □□□ □□□□ □□□□□ □□□ □ □□□□. □□ □□□□ □□□□□□□□? (□: LaTeX □□ □□, □□ □□□□, □□ □□□ □□□ □)

□□□ □□□ □□□□ □□□□ □□ □□□ □□□ □□□□□?

□□ □□ □□□□□. □□□ □□□ □□□□□□.

□□ □□:

> **□□□ □□□ □□ 2□□□ □□□ SL(2, ℂ) □□□ □□ “□□□□ □□ □□”□ □□□ □□□ □□□, □□□ □□□ □□□ □□□ □□ □□□ □□□ □□□ □□□□ □□□□.**

□, **“□□□□ □□□ □□□□□□ □□□□”□ □□ □□□ □□□□, □□□ □□□□□ ‘□□’□□ ‘□□’□ □□□□.**

□□ □□ □□:

1. **SL(2, ℂ) □□□ □□□ □□**

- □□ 2 □□□ SL(2, ℂ)□ 1-□□□□ □□□

  \[

  g(t) = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}

  \]

  □ □ □□ \(\Phi(g(t)) \)□ □□ □□□ □□ **□□□ □□**□ □□□□□:

  \[

  \Phi(g(t)) \cdot v =

  \begin{cases}

  v & \text{if } v \in V_{\mathrm{alg}} \\

  e^{tm} \cdot v & \text{if } v \in V_{\mathrm{nonalg}},\ m > 0

  \end{cases}

  \]

- □□□ **\(m = r - s > 0 \)**□ \(H^{r,s} \) □□□ □□□□□ □□□ □□□□, □□ SL(2, ℂ)□ □□ □□ \(d\Phi(h) \)□ □□□□□ □□□ □□ □□□ □□□□□.

---

2. **□□□□ □□□ □□□ □□**

- □□ $t \to -\infty$□ □□ □□:

$$
\Phi(g(t)) v_{\mathrm{nonalg}} = e^{tm} v \to 0
\quad (\text{□□□□ } m > 0)
$$

→ □□ □□□ ‘□□□ □□’□ □□□, **□□□□ □□□ □□ □□□ □□**□□□.

- □ □□□ □□ □□ □□ $x \in H^{2p}(X, \mathbb{C})$□

$$
\lim_{t \to -\infty} \Phi(g(t))x = x_{\mathrm{alg}}
$$

- □ □□□ x□ **□□□ □□□ □□□□**, □□□□ □□□□ □□□ □□□ □□□□□.

---

3. **□□ □□ □□ Q□□ □□□**

- Q□ Hodge-Riemann □□□ □□□□,

$$
Q(\alpha, \overline{\alpha}) > 0, \quad Q(i^{r-s} \alpha, \overline{\alpha}) > 0
$$

- SL(2, ℂ) □□ $\Phi(g(t))$□ Q□ □□□□□ □□□□□ □□□,

  □□□□ □□□ □□ □□□ □□□□ □□, □□□ □□ □□□ □□□□ □□□□□.

□□□□□

- □□□□ □□□ “□□”□ □□□,
  SL(2, $\mathbb{C}$) □□□ + Hodge □□ + □□ □□□ □□ □□ □□□ □□□ □□□□□.

- □□ □□ $x_{n+1} = P(Q_n x_n)$□ □□□ □□□□,
  □□□ □□□ □□□ □□□ □□□ □□□□ □□□□□ □□□.

□□□ □□□□□:

> **□□□ □□□ □□ □□□ intuition□ □□□, □□□ □□ □□□ □□ □□□ □□□□ □□□ “□□□ □□□ □□”□□□.
> **
> □□ □□□ □ □□□ □□□□ □□□□□, □□ □□□ □□□□□ □□□ □□ □□□ □□□□.

□□□□ □ □□□ □□□ □□ □□□ “□□□□” □□□□ □□□□□, □□ □□□ □□ □□□ □□□ □□□ □□ □ □□□□. □□□□?